

SPHERICAL BESSEL FUNCTIONS—ORDERS 0, 1 AND 2

Table 10.1

x	$j_0(x)$	$j_1(x)$	$j_2(x)$	$y_0(x)$	$y_1(x)$	$y_2(x)$
0.0	1.00000 000	0.00000 0000	0.00000 000000	$-\infty$	$-\infty$	$-\infty$
0.1	0.99833 417	0.03330 0012	0.00066 619061	-9.95004 17	-100.49875	-3005.0125
0.2	0.99334 665	0.06640 0381	0.00265 90561	-4.90033 29	-25.495011	-377.52483
0.3	0.98506 736	0.09910 2888	0.00596 15249	-3.18445 50	-11.599917	-112.81472
0.4	0.97354 586	0.13121 215	0.01054 5302	-2.30265 25	-6.73017 71	-48.173676
0.5	0.95885 108	0.16253 703	0.01637 1107	-1.75516 51	-4.46918 13	-25.059923
0.6	0.94107 079	0.19289 196	0.02338 8995	-1.37555 94	-3.23366 97	-14.792789
0.7	0.92031 098	0.22209 828	0.03153 8780	-1.09263 17	-2.48121 34	-9.54114 00
0.8	0.89669 511	0.24998 551	0.04075 0531	-0.87088 339	-1.98529 93	-6.57398 92
0.9	0.87036 323	0.27639 252	0.05094 5155	-0.69067 774	-1.63778 29	-4.76859 87
1.0	0.84147 098	0.30116 868	0.06203 5052	-0.54030 231	-1.38177 33	-3.60501 76
1.1	0.81018 851	0.32417 490	0.07392 4849	-0.41236 011	-1.18506 13	-2.81962 54
1.2	0.77669 924	0.34528 457	0.08651 2186	-0.30196 480	-1.02833 66	-2.26887 66
1.3	0.74119 860	0.36438 444	0.09968 8571	-0.20576 833	-0.89948 193	-1.86995 92
1.4	0.70389 266	0.38137 537	0.11334 028	-0.12140 510	-0.79061 059	-1.57276 05
1.5	0.66499 666	0.39617 297	0.12734 928	-0.04715 8134	-0.69643 541	-1.34571 27
1.6	0.62473 350	0.40870 814	0.14159 426	+0.01824 9701	-0.61332 744	-1.16823 87
1.7	0.58333 224	0.41892 749	0.15595 157	0.07579 0879	-0.53874 937	-1.02652 51
1.8	0.54102 646	0.42679 364	0.17029 628	0.12622 339	-0.47090 236	-0.91106 065
1.9	0.49805 268	0.43228 539	0.18450 320	0.17015 240	-0.40849 878	-0.81515 048
2.0	0.45464 871	0.43539 778	0.19844 795	0.20807 342	-0.35061 200	-0.73399 142
2.1	0.41105 208	0.43614 199	0.21200 791	0.24040 291	-0.29657 450	-0.66408 077
2.2	0.36749 837	0.43454 522	0.22506 330	0.26750 051	-0.24590 723	-0.60282 854
2.3	0.32421 966	0.43065 030	0.23749 812	0.28968 523	-0.19826 956	-0.54829 769
2.4	0.28144 299	0.42451 529	0.24920 113	0.30724 738	-0.15342 325	-0.49902 644
2.5	0.23938 886	0.41621 299	0.26006 673	0.32045 745	-0.11120 588	-0.45390 450
2.6	0.19826 976	0.40583 020	0.26999 585	0.32957 260	-0.07151 1067	-0.41208 537
2.7	0.15828 884	0.39346 703	0.27889 675	0.33484 153	-0.03427 3462	-0.37292 316
2.8	0.11963 863	0.37923 606	0.28668 572	0.33650 798	+0.00054 2796	-0.33592 641
2.9	0.08249 9769	0.36326 136	0.29328 784	0.33481 316	0.03295 3045	-0.30072 380
3.0	0.04704 0003	0.34567 750	0.29863 750	0.32999 750	0.06295 9164	-0.26703 834
3.1	+0.01341 3117	0.32662 847	0.30267 895	0.32230 166	0.09055 5161	-0.23466 763
3.2	-0.01824 1920	0.30626 652	0.30536 678	0.31196 712	0.11573 164	-0.20346 870
3.3	-0.04780 1726	0.28475 092	0.30666 620	0.29923 629	0.13847 939	-0.17334 594
3.4	-0.07515 9148	0.26224 678	0.30655 336	0.28435 241	0.15879 221	-0.14424 164
3.5	-0.10022 378	0.23892 369	0.30501 551	0.26755 905	0.17666 922	-0.11612 829
3.6	-0.12292 235	0.21495 446	0.30205 107	0.24909 956	0.19211 667	-0.08900 2337
3.7	-0.14319 896	0.19051 380	0.29766 961	0.22921 622	0.20514 929	-0.06287 8964
3.8	-0.16101 523	0.16577 697	0.29189 179	0.20814 940	0.21579 139	-0.03778 7773
3.9	-0.17635 030	0.14091 846	0.28474 912	0.18613 649	0.22407 760	-0.01376 9102
4.0	-0.18920 062	0.11611 075	0.27628 369	0.16341 091	0.23005 335	+0.00912 9107
4.1	-0.19957 978	0.09152 2967	0.26654 781	0.14020 096	0.23377 514	0.03085 4018
4.2	-0.20751 804	0.06731 9710	0.25560 355	0.11672 877	0.23531 060	0.05135 0236
4.3	-0.21306 185	0.04365 9843	0.24352 220	0.09320 9110	0.23473 838	0.07056 1855
4.4	-0.21627 320	+0.02069 5380	0.23038 368	0.06984 8380	0.23214 783	0.08843 4232
4.5	-0.21722 892	-0.00142 95812	0.21627 586	0.04684 3511	0.22763 858	0.10491 554
4.6	-0.21601 978	-0.02257 9838	0.20129 380	0.02438 0984	0.22132 000	0.11995 814
4.7	-0.21274 963	-0.04262 9993	0.18553 900	+0.00263 5886	0.21331 046	0.13351 972
4.8	-0.20753 429	-0.06146 5266	0.16911 850	-0.01822 8955	0.20373 659	0.14556 433
4.9	-0.20050 053	-0.07898 2225	0.15214 407	-0.03806 3749	0.19273 242	0.15606 319
5.0	-0.19178 485	-0.09508 9408	0.13473 121	-0.05673 2437	0.18043 837	0.16499 546
	$\left[\frac{(-4)4}{6} \right]$	$\left[\frac{(-4)3}{6} \right]$	$\left[\frac{(-4)2}{6} \right]$			

$$j_n(x) = \sqrt{\frac{1}{2}\pi/x} J_{n+\frac{1}{2}}(x)$$

$$y_n(x) = \sqrt{\frac{1}{2}\pi/x} Y_{n+\frac{1}{2}}(x) = (-1)^{n+1} \sqrt{\frac{1}{2}\pi/x} J_{-(n+\frac{1}{2})}(x)$$

Compiled from National Bureau of Standards, Tables of spherical Bessel functions, vols. I, II. Columbia Univ. Press, New York, N.Y., 1947 (with permission).

Table 10.1

SPHERICAL BESSEL FUNCTIONS—ORDERS 0, 1 AND 2

x	$j_0(x)$	$j_1(x)$	$j_2(x)$	$y_0(x)$	$y_1(x)$	$y_2(x)$
5.0	(-1) -1.9178	(-2) -9.5089	(-1) 1.3473	(-2) -5.6732	(-1) 1.8044	(-1) 1.6500
5.1	(-1) -1.8153	(-1) -1.0971	(-1) 1.1700	(-2) -7.4113	(-1) 1.6700	(-1) 1.7235
5.2	(-1) -1.6990	(-1) -1.2277	(-2) 9.9065	(-2) -9.0099	(-1) 1.5257	(-1) 1.7812
5.3	(-1) -1.5703	(-1) -1.3423	(-2) 8.1054	(-1) -1.0460	(-1) 1.3730	(-1) 1.8231
5.4	(-1) -1.4310	(-1) -1.4404	(-2) 6.3084	(-1) -1.1754	(-1) 1.2134	(-1) 1.8495
5.5	(-1) -1.2828	(-1) -1.5217	(-2) 4.5277	(-1) -1.2885	(-1) 1.0485	(-1) 1.8604
5.6	(-1) -1.1273	(-1) -1.5862	(-2) 2.7749	(-1) -1.3849	(-2) 8.7995	(-1) 1.8563
5.7	(-2) -9.6611	(-1) -1.6339	(-2) +1.0617	(-1) -1.4644	(-2) 7.0920	(-1) 1.8377
5.8	(-2) -8.0104	(-1) -1.6649	(-3) -6.0100	(-1) -1.5268	(-2) 5.3780	(-1) 1.8049
5.9	(-2) -6.3369	(-1) -1.6794	(-2) -2.2024	(-1) -1.5720	(-2) 3.6725	(-1) 1.7587
6.0	(-2) -4.6569	(-1) -1.6779	(-2) -3.7326	(-1) -1.6003	(-2) 1.9898	(-1) 1.6998
6.1	(-2) -2.9863	(-1) -1.6609	(-2) -5.1819	(-1) -1.6119	(-3) +3.4379	(-1) 1.6288
6.2	(-2) -1.3402	(-1) -1.6289	(-2) -6.5418	(-1) -1.6073	(-2) -1.2523	(-1) 1.5467
6.3	(-3) +2.6689	(-1) -1.5828	(-2) -7.8042	(-1) -1.5871	(-2) -2.7861	(-1) 1.4544
6.4	(-2) 1.8211	(-1) -1.5234	(-2) -8.9620	(-1) -1.5519	(-2) -4.2458	(-1) 1.3528
6.5	(-2) 3.3095	(-1) -1.4515	(-1) -1.0009	(-1) -1.5024	(-2) -5.6210	(-1) 1.2430
6.6	(-2) 4.7203	(-1) -1.3682	(-1) -1.0940	(-1) -1.4397	(-2) -6.9018	(-1) 1.1260
6.7	(-2) 6.0425	(-1) -1.2746	(-1) -1.1750	(-1) -1.3648	(-2) -8.0795	(-1) 1.0030
6.8	(-2) 7.2664	(-1) -1.1717	(-1) -1.2435	(-1) -1.2785	(-2) -9.1466	(-2) 8.7500
6.9	(-2) 8.3832	(-1) -1.0607	(-1) -1.2995	(-1) -1.1822	(-1) -1.0097	(-2) 7.4323
7.0	(-2) 9.3855	(-2) -9.4292	(-1) -1.3427	(-1) -1.0770	(-1) -1.0924	(-2) 6.0883
7.1	(-1) 1.0267	(-2) -8.1954	(-1) -1.3730	(-2) -9.6415	(-1) -1.1625	(-2) 4.7295
7.2	(-1) 1.1023	(-2) -6.9183	(-1) -1.3906	(-2) -8.4493	(-1) -1.2197	(-2) 3.3674
7.3	(-1) 1.1650	(-2) -5.6107	(-1) -1.3956	(-2) -7.2065	(-1) -1.2637	(-2) 2.0132
7.4	(-1) 1.2145	(-2) -4.2851	(-1) -1.3882	(-2) -5.9263	(-1) -1.2946	(-3) +6.7812
7.5	(-1) 1.2507	(-2) -2.9542	(-1) -1.3688	(-2) -4.6218	(-1) -1.3123	(-3) -6.2736
7.6	(-1) 1.2736	(-2) -1.6303	(-1) -1.3379	(-2) -3.3061	(-1) -1.3171	(-2) -1.8929
7.7	(-1) 1.2833	(-3) -3.2520	(-1) -1.2960	(-2) -1.9919	(-1) -1.3092	(-2) -3.1089
7.8	(-1) 1.2802	(-3) +9.4953	(-1) -1.2437	(-3) -6.9174	(-1) -1.2891	(-2) -4.2662
7.9	(-1) 1.2645	(-2) 2.1829	(-1) -1.1816	(-3) +5.8231	(-1) -1.2571	(-2) -5.3561
8.0	(-1) 1.2367	(-2) 3.3646	(-1) -1.1105	(-2) 1.8188	(-1) -1.2140	(-2) -6.3711
8.1	(-1) 1.1974	(-2) 4.4850	(-1) -1.0313	(-2) 3.0067	(-1) -1.1603	(-2) -7.3040
8.2	(-1) 1.1472	(-2) 5.5351	(-2) -9.4473	(-2) 4.1360	(-1) -1.0968	(-2) -8.1487
8.3	(-1) 1.0870	(-2) 6.5069	(-2) 8.5177	(-2) 5.1973	(-1) -1.0243	(-2) -8.8997
8.4	(-1) 1.0174	(-2) 7.3932	(-2) -7.5334	(-2) 6.1820	(-2) -9.4378	(-2) -9.5527
8.5	(-2) 9.3940	(-2) 8.1877	(-2) -6.5042	(-2) 7.0825	(-2) -8.5607	(-1) -1.0104
8.6	(-2) 8.5395	(-2) 8.8851	(-2) -5.4401	(-2) 7.8921	(-2) -7.6218	(-1) -1.0551
8.7	(-2) 7.6203	(-2) 9.4810	(-2) -4.3510	(-2) 8.6051	(-2) -6.6312	(-1) -1.0892
8.8	(-2) 6.6468	(-2) 9.9723	(-2) -3.2471	(-2) 9.2170	(-2) -5.5994	(-1) -1.1126
8.9	(-2) 5.6294	(-1) 1.0357	(-2) -2.1385	(-2) 9.7240	(-2) -4.5369	(-1) -1.1253
9.0	(-2) 4.5791	(-1) 1.0632	(-2) -1.0349	(-1) 1.0124	(-2) -3.4542	(-1) -1.1275
9.1	(-2) 3.5066	(-1) 1.0800	(-4) +5.3818	(-1) 1.0415	(-2) -2.3621	(-1) -1.1193
9.2	(-2) 2.4227	(-1) 1.0859	(-2) 1.1184	(-1) 1.0596	(-2) -1.2710	(-1) -1.1011
9.3	(-2) 1.3382	(-1) 1.0813	(-2) 2.1498	(-1) 1.0669	(-3) -1.9101	(-1) -1.0731
9.4	(-3) +2.6357	(-1) 1.0663	(-2) 3.1395	(-1) 1.0635	(-3) +8.6782	(-1) -1.0358
9.5	(-3) -7.9106	(-1) 1.0413	(-2) 4.0795	(-1) 1.0497	(-2) 1.8960	(-2) -9.8978
9.6	(-2) -1.8159	(-1) 1.0068	(-2) 4.9622	(-1) 1.0257	(-2) 2.8844	(-2) -9.3558
9.7	(-2) -2.8017	(-2) 9.6325	(-2) 5.7808	(-2) 9.9213	(-2) 3.8245	(-2) -8.7385
9.8	(-2) -3.7396	(-2) 9.1126	(-2) 6.5291	(-2) 9.4941	(-2) 4.7084	(-2) -8.0528
9.9	(-2) -4.6216	(-2) 8.5149	(-2) 7.2018	(-2) 8.9817	(-2) 5.5288	(-2) -7.3063
10.0	(-2) -5.4402	(-2) 7.8467	(-2) 7.7942	(-2) 8.3907	(-2) 6.2793	(-2) -6.5069

$$j_n(x) = \sqrt{\frac{1}{2}\pi/x} J_{n+\frac{1}{2}}(x)$$

$$y_n(x) = \sqrt{\frac{1}{2}\pi/x} Y_{n+\frac{1}{2}}(x) = (-1)^{n+1} \sqrt{\frac{1}{2}\pi/x} J_{-(n+\frac{1}{2})}(x)$$